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International Round

2025

The International Mathematical Modeling Challenge (IM²C) Summary Sheet

Global Sports League

Summary

Fair competition in sport events requires a balance of logistics, equity, and practicality, especially when organizing sporting competitions on an international scale. A efficient scheduling system is the basis to achieving such goal. Basketball, a sport played by over 450 million people around the world, is one of the most widely played sport in the world. While current basketball leagues face challenges, including fairness of competition, environmental sustainability and time constraints, there is an urgent need for a more optimized schedule. The schedule should run between 8 and 9 months, include 20 teams on an international scale and consider various factors. In this paper, our team makes a decision that can take all factors into account and determine an optimal time schedule for the upcoming Global Sports League (GSL).

First, **nonlinear programming** is used to determine the objective function and the model. In addition, it is used to find the city most optimal for the final games. According to the model, the optimal place for the games is Boston. **Genetic Algorithms** are then used to sort teams into four groups, with each consisting of stronger teams and weaker teams. This is preparation for round-robin tournaments and then for further elimination stages. The results of the model include graphs of the penalty value and distance, as well as matrix that shows the initial grouping for round-robin. This step ensures the fairness of game.

Moreover, **Analytical Hierarchy Process** is used in our project. We first sent out a survey to more than 30 sport enthusiasts and asked them to rank three factors related to sports from their own perspective. From their choices, we determined the weight to give to each factor while optimizing our project. Lastly, from the above stages, we computed our model. The model consists of grouping for the round robin round, elimination round, semifinals and finals. The initial grouping involves both strong teams and weaker teams, after competing, the team with lowest points are eliminated. Winners continue on to other rounds. Our results show the top three teams are USA (rank 1), Serbia (rank 2) and Germany (rank 3).

We conclude that our model addresses existing issues in basketball. Not only the model is fair for all teams, it is also more economic viable and environmentally friendly, capable of long term usage.

Keywords: Nonlinear Programming, Genetic Algorithm, Sensitivity Analysis, Analytical Hierarchy Process, Sport Scheduling

International Multi-Continental Matchmaking Committee

IMMC
Global Sports League

10th March 2025

To the Directors of the International Multi-Continental Matchmaking Committee (IMMC):

We are pleased to present our proposed scheduling model for the Global Sports League (GSL) basketball games. Our model aligns with the IMMC's goals of geographic diversity, competitive equity, logistical feasibility, and sustainability while ensuring an engaging and fair competition among the 20 participating teams.

Our model ensures representative from all countries. Representatives are chosen based on countries population. The country with the most population, Asia has five representative teams. While Africa has four teams, Europe, South America and North America has three teams, Oceania has two teams. Later, representative countries within each continent were chosen based on the FIBA world ranking. The highest ranking teams in each country was chosen. This fosters local rivalries and reducing logistical challenges by minimizing excessive travel within regions. In addition, the round robin game guarantees fairness by balancing home and away games, providing equal opportunities for all teams to compete under similar conditions.

Through this model, we hope to minimize travel distance, maximize competition equality and minimize carbon dioxide emissions. We completed this process through conducting a survey to 30 sport enthusiasts. By analyzing the results, we gave weight to different factors. Later, by running algorithms, we sorted different teams into groups that consist of stronger and weaker teams. This was the initial sorting for the round robin game. As teams compete between 2 week intervals, winning teams will advance into elimination rounds, and finally, determine the winner. To determine the optimal place for competitions after round robin, we used the longitude and latitude of each capital to calculate which place is fair for all teams. The nearest city who has the ability to host the event was Boston, USA. This approach reduces travel distances and associated costs, aligning with global sustainability goals and maintaining fairness during competitions. By scheduling games through this approach, we maximize global audience reach and ensure competition equity. Overall, our model supports the IMMC's objectives by balancing inclusivity, fairness, and practicality, setting a new standard for international sports leagues.

In comparison to other models, our model displayed is a more comprehensive model, fulfilling all the criteria of minimal carbon emission, minimizing travel distance, maximizing resting round, minimizing group variance and maximizing attractive index. While some models might excel in one of the criteria and ignoring the other, our model can address all the criteria to a high extent.

Team 25021172

GAME SCHEDULE

Group Round Robin Game Schedule

Apr 1st-Apr 14th

HOME		VS.		AWAY
		VS.		
		VS.		
		VS.		

Apr 15th-Apr 28th

HOME		VS.		AWAY
		VS.		

Apr 29th-May 12th

HOME		VS.		AWAY
		VS.		
		VS.		

May 13th-May 26th

HOME		VS.		AWAY
		VS.		
		VS.		
		VS.		
		VS.		

May 27th-June 9th

HOME		VS.		AWAY
		VS.		

Jun 10th-Jun 23th

HOME		VS.		AWAY
		VS.		
		VS.		
		VS.		
		VS.		

Jun 24th-Jul 7th

HOME		VS.		AWAY
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Jul 8th-Jul 21th

HOME		VS.		AWAY
		VS.		
		VS.		
		VS.		
		VS.		
		VS.		

Jul 22th-Aug 4th

HOME		VS.		AWAY
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Aug 5th-Aug 18th

HOME		VS.		AWAY
		VS.		
		VS.		
		VS.		
		VS.		
		VS.		

Elimination Round Game Schedule

Aug 19-Sep 1

	
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Sep 2- Sep15

	
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Sep 16- Sep 29

	
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Sep 30- Oct 13

	
---	---

Oct 14-Oct 27

	VS.	
---	-----	---

Oct 28- Nov 10

	VS.	
--	-----	---

	Bronze Medal Match	
	Nov 11-Nov 24	

	Champion Match	
	Nov 11-Nov 24	

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1 Introduction

Fair competition in sports events requires a balance of logistics, equity, and practicality, particularly when organizing international tournaments. An efficient scheduling system serves as the foundation for achieving this goal. Currently, international basketball competitions face challenges related to competitive fairness, such as ensuring equal rest and recovery time by minimizing back-to-back games, optimizing travel distance, adhering to time constraints, and maintaining a balanced distribution of home and away games. Given these complexities, there is an urgent need for a scheduling system that is geographically diverse, competitively fair, logistically feasible, and both economically and environmentally sustainable.

In this paper, we develop a non-linear programming model to address these objectives. Using the analytical hierarchy model, we evaluate key factors such as travel distance, competition variance, interval variance, carbon emissions, and commercial attractiveness. This step aims to match teams with similar rankings, minimize travel distance, maximize rest time, and enhance commercial value. We then implement genetic algorithms to optimize the model. Finally, we generalize our approach and assess the effectiveness of the proposed scheduling system. Our model ensures that international basketball competitions with 20 teams are scheduled fairly, meeting both logistical and competitive requirements.

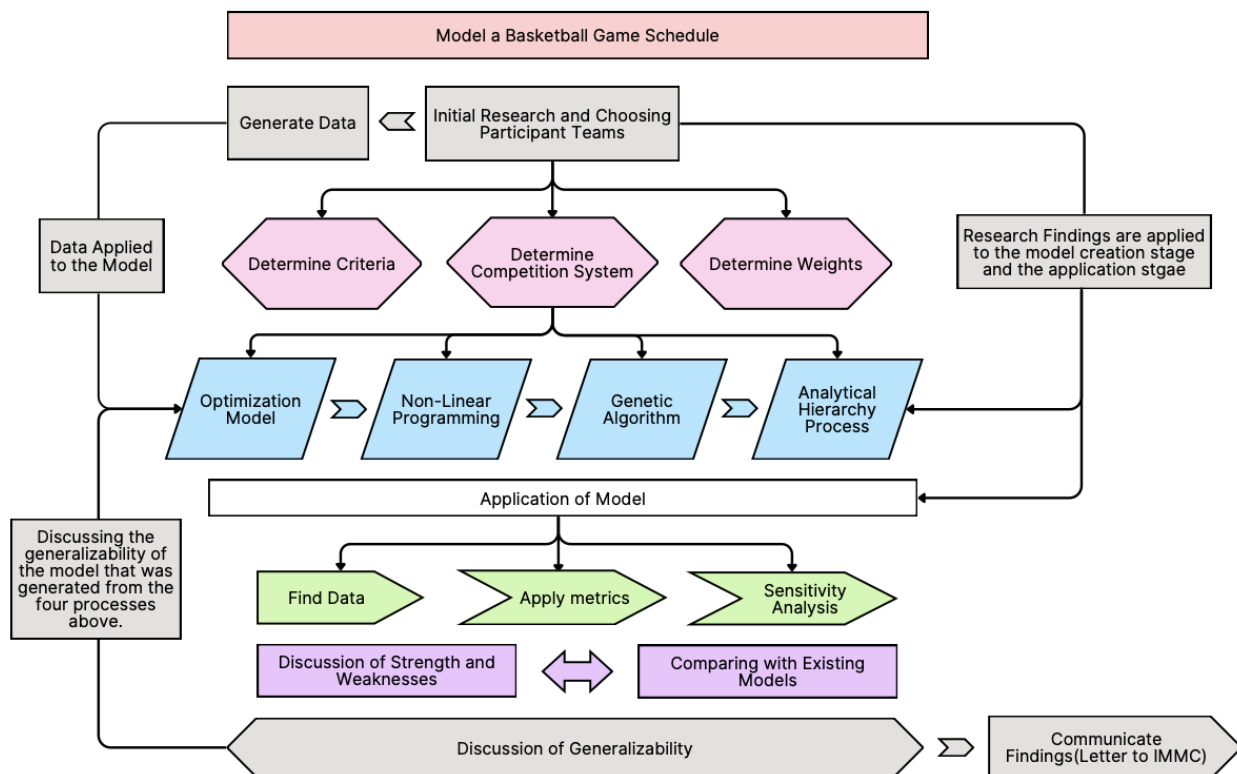


Figure 1 Flow Chart of Process

2 Preliminary Definitions and Assumptions

2.1 Definitions

Table 1 Variable definitions

Notations	Descriptions
i	The number code of each team. The order of the codes are generated by the continents
j	The number code of the opponent team for the home team
k	The number of rounds of the game
g	The number code of current group playing
w	The competitiveness level of each group
c	The carbon emission every kilometer
L	The competitiveness level of each group
G	Number of groups of the competition
T	The total number of teams participating in a tournament
y_{ijk}	Team i plays team j at home in round k
y_{jik}	Team j plays team i at home in round k
x_{ig}	Team i being assigned to group g
s_i	The competitiveness score of team i
s_g	The competitiveness of a particular group that participates in the competition
d_{ig}	The distance between team i and team j when team i is the home
G_L	The overall competitiveness of G participant teams.
(lot_i, lat_i)	The latitude and longitude coordinates of team i

2.2 Assumptions

Assumption 1: The games would not be canceled or affected due to random events. This assumption ensures that the game schedule will not be shifted or changed due to random factors such as weather conditions that cause the games to stop or natural disasters causing the game to take place in other cities. The schedule is solely focused on the order and result of each game.

Assumption 2: Teams adapt to different venues equally well. This assumption ensures the fair and just outcomes of the game. The effect of adapting to sports arenas, tiredness of traveling, and other factors are not considered.

Assumption 3: The time between the beginning of the competition and the beginning of a team's first game and the time between the last game of a team in the competition before it ends does not influence the performance of the team. This assumption is created to avoid the situation that teams that start their first game later have more time to train compared to the ones who start their first game earlier.

Assumption 4: Player's physical exertion is equal in each game and each team. This assumption is necessary because it allows the schedule only to focus on optimizing the best game that fulfills the requirements of the GSL and IMMC. This assumption also ensures that no team is systematically disadvantaged due to unpredictable or uncontrollable factors, promoting consistency and competitive balance.

Assumption 5: The representatives of each continent are members that have the highest ranking and score in the FIBA list. This assumption is created because we believe if the con-

tinents are choosing the representative teams, they will be choosing the most high-ranking teams.

Assumption 6: The competition must be completed in this year, and it takes place from April to December in this year. It is required that the competition should be completed within eight to nine months.

3 Developing the Model

3.1 Identifying Subject of Study

3.1.1 Initial Research

As many sports events have clearly pronounced official rules for grouping, scheduling and playing, a model developed for sport scheduling should strictly adhere to the limitations posed by the official organization for the specified sport. Therefore, relevant data and resources consulted in this research mainly originated from official sites, such as the International Basketball Federation (FIBA), the International Federation of Association Football etc. While other relevant data were retrieved from governmental accounts or large authoritative institutions. A complete list of data sources is listed as follows:

Table 2 Data Sources and Uses

Information	Source	Site
Different types of game formats for sports	National Collegiate Athletic Association	https://www.ncaa.com
Game formats previously used	International Olympics Committee	https://www.olympics.com
Current rules, ranking, and scores of male Basketball teams	International Basketball Federation	https://www.fiba.basketball/en
Current rules, ranking, and scores of male Football teams	International Federation of Association Football	https://www.fifa.com
Geographical data	Google Maps	https://www.google.com/maps
Demographics of continents	United Nations Department of Economic and Social Affairs	https://www.un.org/development/desa/dpad/
Carbon emissions data	Government of the United Kingdom	https://www.gov.uk

- Data collected is used to define a choice of sport for this Global Sport League (GSL) tournament, as well as the criteria for choosing participant countries in the games.
- Research highlights 5 decisive factors influencing the optimal scheduling plan: representativeness of different continents, balanced competitiveness, fair allocation of home

games and away games, travel distances between competing teams, and resting intervals between games to be incorporated in the scheduling model.

3.1.2 Choice of Sport

Basketball is a dynamic team sport played between two teams each consisting of five players. [1] The primary goal of the game is to score by shooting the ball through the opponents hoop. Players can move the ball through dribbling or passing, and the team with the most points win the match. Today, basketball has grown to become one of the most widely played sport in the world, with at least 450 million people worldwide. Moreover, international basketball games are among the favorites of sport enthusiasts. In the international tournaments, it is especially important to consider factors such as fairness, distance each team has to travel and the most optimal amount of time between games. Current international basketball leagues face problems such as competitive fairness, travel optimization and time constraints. Thus, to ensure competition equality and better game experience, an optimized schedule is especially vital.

3.1.3 Choice of Teams

We determined the number of teams chosen to represent a continent by referencing the country's population. [2]

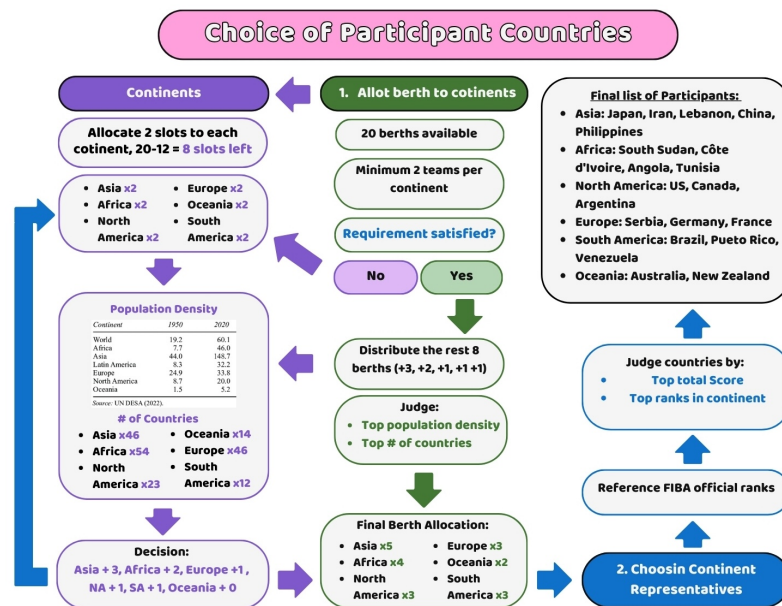


Figure 2 Flowchart: Choice for participating countries

When choosing participant teams, we started from allotting continents into 20 berths. Each continent has a minimum amount of two teams. According to their population ratio(continent population/world population) and population density, another one to three team were added to each continent accordingly. Thus, the final berth allocation of teams are

Table 3 List of Chosen Countries

Country	Continent	Ranking	PTS value	Capital	Location
Japan	Asia	21	528.5	Tokyo	35.6895° N, 139.6917° E
Iran	Asia	28	414.1	Tehran	35.6892° N, 51.3890° E
Lebanon	Asia	29	409.4	Beirut	33.8938° N, 35.5018° E
China	Asia	30	397.4	Beijing	39.9042° N, 116.4074° E
Philippines	Asia	34	377.1	Manila	14.5995° N, 120.9842° E
South Sudan	Africa	23	488.5	Juba	4.8594° N, 31.5713° E
Cote d'Ivoire	Africa	31	392.5	Yamoussoukro	6.8276° N, 5.2893° W
Angola	Africa	33	382.2	Luanda	8.8300° S, 13.2450° E
Tunisia	Africa	36	339.2	Tunis	36.8065° N, 10.1815° E
Serbia	Europe	2	761.1	Belgrade	44.7866° N, 20.4489° E
Germany	Europe	3	757.3	Berlin	52.5200° N, 13.4050° E
France	Europe	4	753.4	Paris	48.8566° N, 2.3522° E
USA	NA	1	840.1	Washington DC	38.9072° N, 77.0369° W
Canada	NA	6	744.9	Ottawa	45.4215° N, 75.6972° W
Puerto Rico	NA	15	606.9	San Juan	18.4655° N, 66.1057° W
Argentina	SA	8	732.5	Buenos Aires	34.6037° S, 58.3816° W
Brazil	SA	12	672.9	Brasilia	15.7942° S, 47.8822° W
Valenzuela	SA	25	459.6	Caracas	10.4806° N, 66.9036° W
Australia	Oceania	7	733.1	Canberra	35.2809° S, 149.1300° E
New Zealand	Oceania	22	492.4	Wellington	41.2924° S, 174.7787° E

determined. To choose teams from each continent, we referenced the FIBA world basketball ranking, and top total score and top ranking teams of the continent.

3.2 Model Overview

3.2.1 Group Round Robin Stage

Group round robin game schedules were planned and determined based on the FIBA basketball ranking. After the 20 countries were chosen, they were sorted into 5 groups, with group 1 being the strongest and group 5 being the weaker teams. Then, our target was to put in equal amount of teams into four groups, where each group contains stronger teams and weaker teams and that each group has teams from multiple countries. Thus, we created an optimization model to sort the teams into groups. The round robin game refers to a game where each contestant meets every other participant in turn. The teams were put into different weeks to compete, and the winners are the teams with the most wins (most points). The winners of each group will then be qualified into the top 8 teams, and then the semifinals and finals.

3.2.2 Elimination Stage

The location we chose for the match of the top 8 teams, semifinals and finals is in Boston, United States. By using the latitude and longitude of the capital city of each country, we

found that the most optimal place is in latitude 49.525 and longitude -47.15, located in the Atlantic Ocean. Thus, we looked for the nearest cities beside. The nearest option was Halifax, Nova Scotia in Canada. However, considering that Halifax have not hosted international basketball games previously, or have stadiums enough to fit in all international teams. Another suitable city, in this case, was Boston, Massachusetts in the USA which was 1600km from the most optimal point. Not only it has previously hosted international basketball games, and also has large stadiums such as the TD garden. Therefore, we decided that all of the last few games will be hosted there.

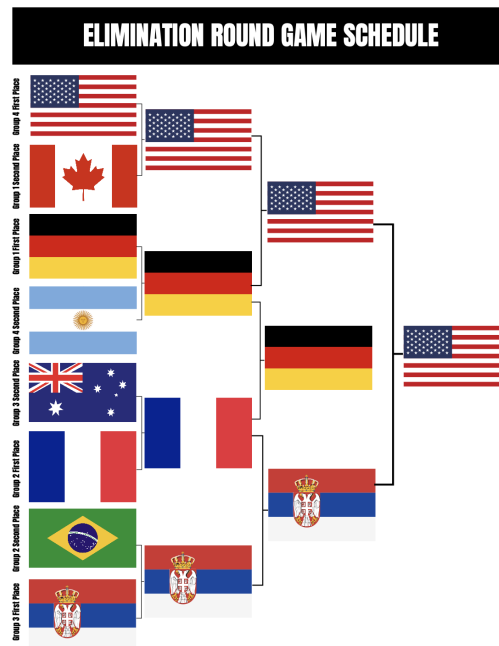


Figure 3 Elimination Stage Game Schedule



Figure 4 Map for Venue of Elimination Stage

3.3 Non-Linear Programming for Initial Scheduling

Nonlinear programming is a technique focused on solving problems where the objective function or constraints are nonlinear. This often includes finding solutions in accordance to the constraints give, and thus, determine the optimal solution.

3.3.1 Objectives

- **Objective 1: Ensuring Fair Distribution in Grouping**

The model aims to minimize the competitiveness variance between different teams, and intends to make the allocation of the teams within each group fair to each team.

$$\min \sum_{g=1}^G \left(\sum_{i \in T} *x_{ig} - \frac{1}{G} \sum_{1 \in T} s_i \right)^2 \quad (1)$$

where G represents the number of groups of the competition, g represents the current group number, i represents the number code of the team, T represents the total number of team participated in the competition, x_{ig} represents team i is currently in group g , s_i represents the competitiveness of team i

- **Objective 2: Minimize travel distances**

The model is required to be minimizing the distance between teams in each game.

$$\min \sum_k \sum_{i,j} d_{ij} * y_{ijk} \quad (2)$$

where k represents the number of rounds played in the game, i represents the number code of the team, j represents the number code of the opposing team. d_{ij} represents the distance between team i and team j when team i it the home, y_{ijk} represents team i playing against team j at home in round k

- **Objective 3: Fair Distribution of Schedule** Distribution fairness refers to minimizing the different in the intensity of games in the weekly schedule and avoid the situation of consecutive games . Our model subtracts the maximum number of games a team plays each week and subtract it with the minimum number of games to minimize the variance of intensity for each team every week.

$$\min \sum_i \left(\max_k \left(\sum_{j \in T} y_{ijk} + y_{jik} \right) - \min_k \left(\sum_{j \in T} y_{ijk} + y_{jik} \right) \right) \quad (3)$$

where i represents the number code of the team, k represents the number of rounds of the game, j represents the number code of the opposing team, T represents the total number of teams participated in the competition, y_{ijk} represents team i playing against team j at home in round k , y_{jik} represents team j playing against team i at home in round k .

Finalize Objective:

We sent out a survey to 30 sport enthusiasts and asked them to rank the three objective factors related to sport from their own perspective. The three factors referred to the three objectives, which are the importance of equitable distribution of teams (including representation of different countries, distribution of intensity, etc.)(represented by A); minimization of travel cost between teams(represented by B); and the equitable distribution of fixtures (e.g., reducing the number of consecutive high-intensity matches, spacing out high-intensity matches, etc.)(represented by C) . From the survey result, we learned that most participants believe the importance of factor B is one half of the importance of factor A, and the importance of factor C is five forth the importance of factor A. Thus, most participants believe that C is the most important factor, followed by factor A, then factor B. Thus, while completing the AHP model, we set the weight according to the survey result.

$$\min \sum_k \sum_{i,j} d_{i,j} + \lambda_1 * \sum_i \left\{ \max_k r_{ik} - \min_k r_{ik} \right\} + \lambda_2 * \sum_{g=1}^G \left(\sum_{i \in T} x_{ig} - \frac{1}{G} \sum_{i \in T} s_i \right)^2 \quad (4)$$

This function binds the three functions above that each represent Fair Distribution, Minimize Travel Costs, and Fair Distribution of Schedule. Specifically, as the third objective function's weight is defined as 1, the other two objective functions are timed to a determined weight. In the final objective function, we aimed to minimize the result of the three functions.

where k represents the number of rounds of the game, i represents the number code of the team, j represents the number code of the opposing team, λ represents the weight of each equation, r_{ik} represents, y_{ijk} represents team i playing against team j at home in round k , y_{jik} represents team j playing against team i at home in round k , T represents the total number of teams participated in the competition.

3.3.2 Constraints

- **Each team is only assigned to one group**

In the group competition stage, each team is required to be only assigned to one group. This ensures that the team is able to play against each team within their group, and possibly become the first two place in its group to compete in the elimination stage.

$$\sum_g x_{ig} = 1 \quad (5)$$

where g represents the current group number, x_{ig} represents team i is currently in group g .

- **Each group can only be assigned five teams**

Since there are 20 teams in total, and one team cannot be involved in more than one group, each group is only allowed to have 5 teams in it. Thus, the total amount of teams in a group is 5.

$$\sum_i x_{ig} = 5 \quad (6)$$

where i represents the number code of the team, x_{ig} represents team i is currently in group g .

- **Each group must include teams of different competitiveness**

The teams are being divided into five different competitiveness rates, according to the FIBA world basketball ranking list.

$$\sum_{i \in G_L} x_{ig} \geq 1, \forall L = 1, 2, 3, 4, 5 \quad (7)$$

where i represents the number code of the team, G_L represents the division of the team in its group, x_{ig} represents team i is currently in group g , L represents the distance between the represented country and the game venue.

- **Competitions are only held within the group in the group round robin stage**

Top two teams in each group are allowed to compete in the elimination stage, thus in the round robin stage, competitions are only allowed to compete within their group.

$$y_{ijk} = 0 \quad \text{if } x_{ig} * x_{jg} = 0 \quad (8)$$

where y_{ijk} represents team i playing against team j at home in round k , x_{ig} represents team i is currently in group g , x_{jg} represents team j is currently in group g .

- **Teams need to play an equal number of home and away games**

To ensure fairness, the number of home games played by each team should be equal to the number of away games played.

$$\sum_{j \in T} y_{ijk} = \sum_{j \in T} y_{jik} \quad (9)$$

where j represents the number code of the opposing team, y_{ijk} represents team i playing against team j at home in round k , y_{jik} represents team j playing against team i at home in round k .

- **One team can only play against the other team once**

The maximum number of games between the same two teams cannot exceed 1.

$$\sum_{j \in T} y_{ijk} + \sum_{j \in T} y_{jik} \leq 1, \forall k \in \{1, 2, 3, \dots, K\} \quad (10)$$

where j represents the number code of the opposing team, y_{ijk} represents team i playing against team j at home in round k , y_{jik} represents team j playing against team i at home in round k , k represents the number of rounds of the game.

- **Avoid Continuous competition**

To ensure fairness components, the model is required to avoid three continuous home games or away games. Each team should not exceed 2 or more continuous games.

$$\sum_{k=w}^{w+2} \sum_{j \in T} y_{ijk} \leq 2 \quad (11)$$

where w represents the current round of the game, j represents the number code of the opposing team, T represents the total number of teams participated in the competition, y_{ijk} represents team i playing against team j at home in round k .

$$\sum_{k=w}^{w+2} \sum_{j \in T} y_{jik} \leq 2 \quad (12)$$

where where w represents the current round of the game, j represents the number code of the opposing team, T represents the total number of teams participated in the competition, y_{jik} represents team j playing against team i at home in round k .

- **Rest after long-term games**

To ensure fairness, each team is required to rest for at least one week after long-term games.

$$y_{ijk} + y_{ij(k+1)} \leq 1 \quad \text{if } d_{ij} \geq d_0 \quad (13)$$

where y_{ijk} represents team i playing against team j at home in round k , d_{ij} represents the distance between team i and team j when team i is the home.

- **Special Occasions**

Considering real life situations, a team cannot play against its self.

$$i \neq j \quad (14)$$

where i represents the number code of the team, j represents the number code of the opposing team.

3.4 Genetic Algorithm

3.4.1 Introduction to Genetic Algorithm

The method used to solve the non-linear programming problem is the **Genetic Algorithm (GA)**. [?]Utilizing a heuristic search technique, the genetic algorithm simulates the process of evolution and natural selection to find the optimal solution to a provided problem. The explicit steps of this process are listed subsequently:

1. **Initialization:** From all potential solutions, an initial population is generated in the form of "DNA" - a binary sequence encompassing all mandatory input variables. Individual DNA strings encode the constraint parameters for all variables subject to change.
2. **Conversion:** A transformation is made to convert the attained binary sequences to decimal variables which is fitted across the full range of possible data.
3. **Selection:** Evaluation of each "DNA"'s "fitness" is conducted in this step. The "fitness" represents its rate of success in solving the objective function. The higher a sequence's "fitness" is, the higher the possibility of "surviving and reproducing" this sequence will have.
4. **Reproduction:** This operation is done by emulating the crossover, which refers to the process of two arbitrarily paired parents exchanging segments of DNA at random sites.
5. **Mutation:** Mutations occur during this process, where DNA fragments of a child are randomly chosen and modified, thus creating variation in the population.
6. **Repetition until convergence:** The aforementioned operations are repeated for an indicated number of generations, or unlimitedly until an optimum is discovered.

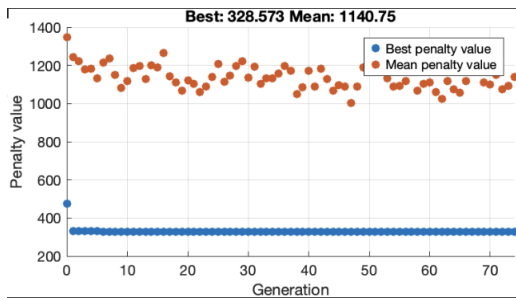
Given its ability to navigate in large, complex solution spaces and effectively find the global optimum, the genetics algorithm is particularly compatible with nonlinear problems. An exemplification of this algorithm is shown as pseudocode in algorithm 1:

3.4.2 Genetic Algorithm in Our Model

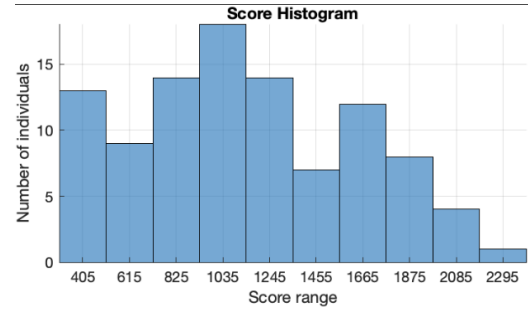
Teams are then sorted into four groups, each group containing teams with higher points and teams with lower points to ensure fairness. The groups are sorted based on Objective 1. After being sorted into groups, the teams will play a round-robin tournament. That is, teams compete against every other team in the group; each contestant faces off against the other contestant in turn. After group stage, the teams with the lowest points will be eliminated, while the two teams with the highest points (most wins) will be qualified for the semi-finals and the finals.

4 Model Results

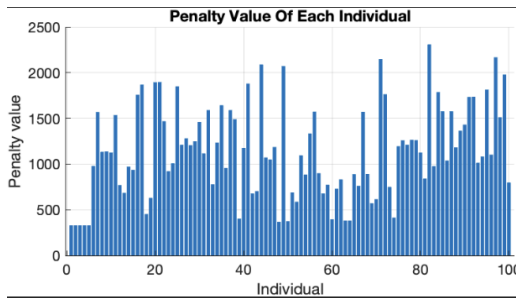
The winning teams according to our model results were, USA (1st place), Serbia (2nd place) and Germany (third place). This is the same as the FIBA ranking, suggesting the accuracy of our model.



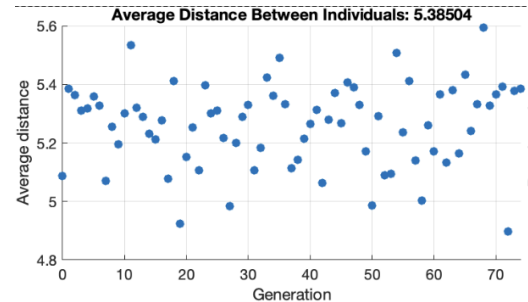
(a) Penalty Value



(b) Score Histogram



(c) Penalty Value of Each Individual



(d) Average Distance Between Individuals

Figure 5 Results of Genetic Algorithm

5 Extend the Model

5.1 New Teams Included

Four teams are added to the 4 allocated groups to prove the flexibility of our model. Specifically, we aim to increase the diversity of teams across continents, so we chose to choose teams from North America, South America, Africa, and Europe. Specific countries from each continent are then picked according to the FIBA basketball ranking, and being distributed into the four separate groups.

Table 4 New Teams Included

Spain	Europe
Latvia	Europe
Dominican Republic	North America
Uruguay	South America

5.2 Changed Constraints

Since four more teams selected from diverse continents are added to the original 20 teams, while most of the constraints remain the same, several constraints are changed. The changes are as follows:

- One team cannot be involved in more than one group

In the original model, the top two member from each group can advance into the Final Eights for the eliminations, meaning a total of

$$\frac{8}{4} = 4 \quad (15)$$

groups are needed. Thus, the 20 teams in the original model is divided into four groups, leaving:

$$\frac{20}{4} = 5 \quad (16)$$

teams to be allocated into each group.

With the 4 new teams added, there are now 24 teams in total, thus, the number of teams that would be arranged into each team would become:

$$\frac{24}{4} = 6 \quad (17)$$

As the constraint in the original model asked that one team cannot be involved in more than one group, each group is now allowed to have 6 teams inside. Thus, constraint would be changed to:

$$\sum_i x_{ig} = 6 \quad (18)$$

- **Each group must include teams of different competitiveness** In the original model with 20 teams, the teams were divided into 5 competitiveness levels, each including 4 teams, so that there could be one team from each level in each group. As the number of teams in the new model included 4 more teams, there will be 1 more team in each group. Thus, there need to be one more level. So that each of the 6 levels contain 4 teams, allocating each team into one of the four groups.

$$\sum_{i \in G_L} x_{ig} \geq 1, \forall L = 1, 2, 3, 4, 5, 6 \quad (19)$$

- **Teams from the same continent should be distributed to different groups:** To ensure fairness components, the model is required to avoid three continuous home games or away games. The team should not have more than three continuous games in this circumstance.

$$\sum_{k=w}^{w+3} \sum_{j \in T} y_{ijk} \leq 3 \quad (20)$$

where w represents the current round of the game, j represents the number code of the opposing team, T represents the total number of teams participated in the competition, y_{ijk} represents team i playing against team j at home in round k .

$$\sum_{k=w}^{w+3} \sum_{j \in T} y_{jik} \leq 3 \quad (21)$$

where w represents the current round of the game, j represents the number code of the opposing team, T represents the total number of teams participated in the competition, y_{jik} represents team j playing against team i at home in round k .

- **Home and Away ratios** Since new teams are added, the difference between the home game and away games of the opposing two teams should be less than 1.

$$\sum_{j \in T} y_{ijk} - \sum_{j \in T} y_{jik} = 1 \quad (22)$$

where j represents the number code of the opposing team, T represents the total number of teams participated in the competition, y_{jik} represents team i playing against team j at home in round k , y_{ijk} represents team j playing against team i at home in round k .

6 Evaluation of Model

6.1 Evaluation Criteria

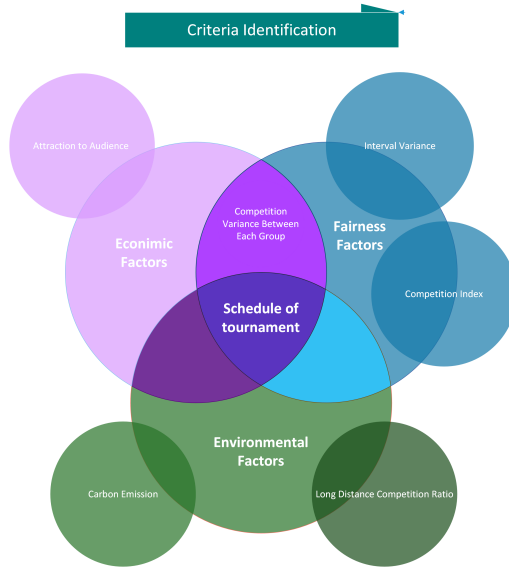


Figure 6 Criteria Identification

1. Long Distance Competition Ratio

Long Distance Ratio is calculated by the number of long distance games dividing the total number of games played, bigger the ratio is, the more long distance games are played.

$$\text{travel distance} = \frac{\text{Number of long distance games}}{\text{Total number of games played}} \quad (23)$$

2. Competition variance between each group

Competition variance is calculated by timing the sum of competitiveness variance between a particular group and the average rate with one over the number of participating groups.

$$\text{Competition Variance} = \frac{1}{G} \sum (s_g - \bar{s})^2 \quad (24)$$

where G represents the number of groups in the game, S_g represents the competitiveness of a particular group that participates in the competition, $\bar{S}$ represents the average competitiveness of the teams.

3. Total Number of Resting Rounds

Total Number of Resting Roundss is calculated by finding the sum of the time interval between each game, and calculate the variance of it.

$$\text{Total Number of Resting Rounds} = \sum_i \sigma_i^2 \quad (25)$$

where σ_i^2 represents the variance of the time interval between each game for team i .

4. Competition Index

Competition Index is calculated by dividing the variance of competitiveness of a particular team's index within its group with the average competitiveness index of this team within a group.

$$\text{Competition Index} = \frac{\text{Variance of } s_i \text{ index within a group}}{\text{Average } s_i \text{ index within a group}} \quad (26)$$

where s_i represents the competitiveness of team i .

5. Total Carbon Emission

Total Carbon Emission can be calculated by the sum of the distance between two opposing teams, timing the number of players in each team(12 players for basketball teams), timing the carbon emission of airplanes every kilometer.

$$\text{Total Carbon Emission} = \sum d_{ij} * 12 * c \quad (27)$$

where d_{ij} represents the distance between team i and team j when team i is the home team, c represents the carbon emission every kilometer.

6. Attraction to audience

Attraction to audience is calculated through the sum of rounds between attractive teas, which are games played between high ranking teams, times the sum of rounds between all teams.

$$\text{Attraction} = \sum_{i,j \in Top} \sum_{s_i} y_{ijk} \quad (28)$$

where i represents the number code of the team, j represents the number code of the opposing team, s_i represents the competitiveness of team i , k represents the number of rounds of the competition, y_{ijk} represents team i playing against team j at home in round k .

6.1.1 Evaluation Results

Table 5 Sensitivity Analysis Result

Criteria	Round Robin	Group Competition	Olympics	Our Model
Total number of single round robin matches	190	75	25	47
Total traveling distance(km)	1655288	442880	35256	379480
Total Carbon Emission(CO ₂ /kg)	4886400	1120200	104080	1120200
Average Competition Index	4.5311	0.3021	0.3932	0.3021
Group Variance	277.8668	183.549	60.1745	82.1434
Resting round	0	14	3	6
Attractiveness Index	0.0526	0.133	0.44	0.1277

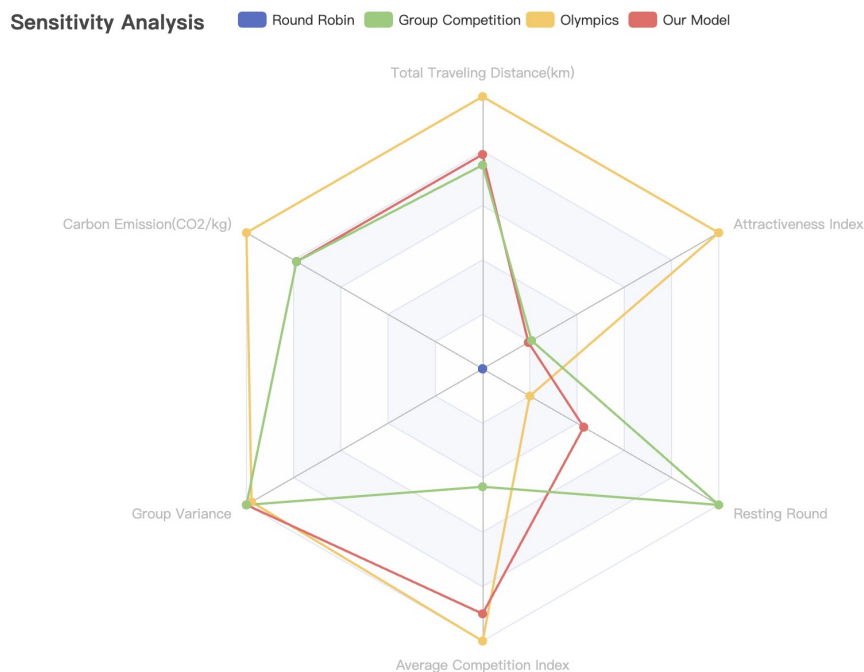


Figure 7 Radar Graph for Sensitivity Analysis Results

We compared our model to three other existing basketball games, which are the round robin, group competition and the Olympics.

1. The first factor we considered is the total traveling distance (km), the smaller the distance the better. From the graph, our model is among one of the best model in minimizing traveling distance (379480).

2. The second factor considered was the total carbon emissions while attending each competition. The lesser the distance needed to travel means that there will be less carbon dioxide emitted into the atmosphere. As our model minimizes travel distance to a full extent, it is also one model that has the least amount of CO₂ emissions (1120200) compared to the Olympics, group competition and round robin.

3. The third factor was the average competition index. The model with the smallest index is the best. According to the graph, our model has a small value compared to the Olympics

or round robin, but not with the group competition. This might be a part where we would advance in the future.

4. The fourth factor is group variance, smaller values mean it is better. All four models have similar values, our model (82.1434), Olympics (60.1745), group competition (183.549) and round robin (277.8668). This shows that we are amongst the best model that optimizes group variance.

5. The fifth factor evaluated was the resting round. More resting rounds mean that athletes can relax and gain more energy. Data show that group competition offer the most resting time out of all (14). We are among the third place (6) and would advance in this part later.

6. The last factor we brought in contrast is the attractiveness index (number of matches played between popular teams). Higher attractive index means that more audience are likely to attend the competition and thus, earn more money. Our model is put in the second place when comparing to the other models. The Olympics, as the most well known international sport game, features the most attractiveness index.

Overall, our model supports the IMMC's objectives by balancing carbon emissions, travel distance, average competition index and group variance, setting a new standard for international sports leagues.

6.2 Sensitivity analysis

The parameters referenced in the model cannot be assumed as fixed and unchanged, in a real-world scenario, fluctuations and changes with these parameters vary greatly depending on the situation. Hence, sensitivity analysis is required to assess the degree of uncertainty, and identify the parameters that have the strongest influence on the results. This information can then be used to analyze the adaptability of the model when key parameters are changed, helping with the refinement of the model and the optimization of parameters.

Three main parameters were tested for sensitivity analysis:

1. The total number of rounds
2. The distances between team locations
3. The score/rating of each team

In the sensitivity analysis, the current parameters were defined as 1, or 100%. Two parameter setting methods are constructed. The parameters were designated to fluctuate within a percentage range. The first variable increased by a 10% step, while both other variables increased by 5% steps. The Genetic algorithm was repetitively applied to the problems with every change in the parameters.

Parameter Variations		
Indicator variable	Value fluctuation threshold (%)	Range (%)
1	-30 to 30	60
2	-10 to 10	20
3	-10 to 10	20

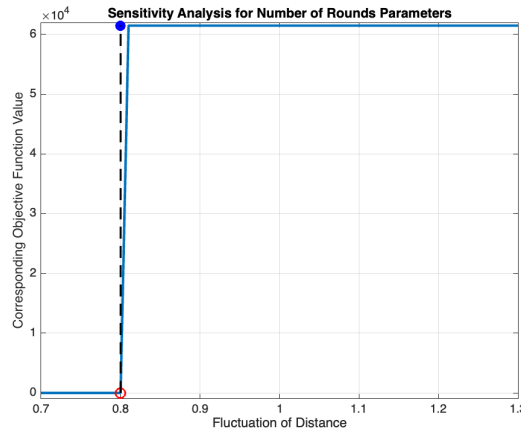


Figure 8 Change in total number of rounds

6.2.1 Analysis on changes of the total number of rounds

The sensitivity analysis graph for numbers of rounds displays a piece-wise function, with the entire first part = 0 and the second part having a slope of 0, staying constant. A jump discontinuity can be found at the adjoint of the two sections, $x = 0.8$, indicating the minimum number of rounds allowed is 8.

This is a reasonable phenomenon. There are 5 teams in each group, and each team needs to play 4 games in the round robin format.

However, one constraint of the function forbids the occurrence of 2 consecutive games by one team, demanding at least 1 round of rest time between each game. Meaning each team can only play:

$$\frac{k}{2} \quad (29)$$

games for k rounds. When k is equal to or greater than 8, the games a team can play in a total of k rounds will always exceed 4, the number of games a team need to play in a group of 5. However, when k is less than 8, the total number of games a team can play in k rounds will be smaller than the minimum amount of games a team need to play in the round robin stage. Therefore, as long as the number of teams in a group remains 5, the function has no viable solutions when the parameter k is smaller than 8. Explaining the jump discontinuity in the graph.

6.2.2 Analysis on changes of distances between team locations

The assumption in the current model is that the games will always be hosted at the capital of a city. Yet, oftentimes, the capital of a country may not be the location where sports enthusiasm is the highest. For instance, in the U.S., the best location for a basketball tournament might not be in Washington D.C., but in Los Angeles. Therefore, the host location of the games is a variable parameter in the model, and changes in the host location will simultaneously change the distance between the teams. Thus, how fluctuations of distance affected the model was tested, yielding the following results for each group:

The trends are directly proportional, as the objective function value aims to minimize the total distances traveled by the group, the larger the distance between the teams, the larger the objective function will be.

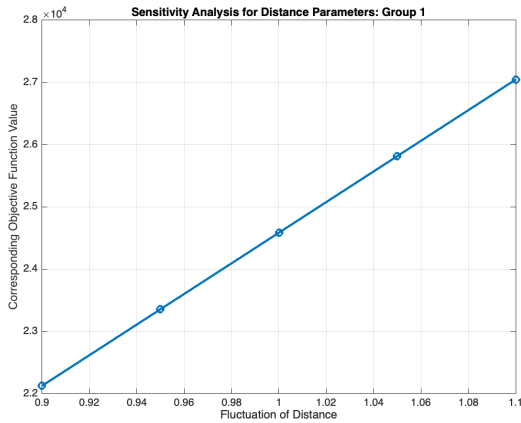


Figure 9 Sensitivity analysis for Group 1

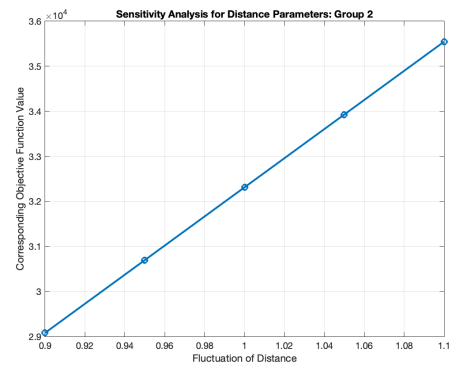


Figure 10 Sensitivity analysis for Group 2

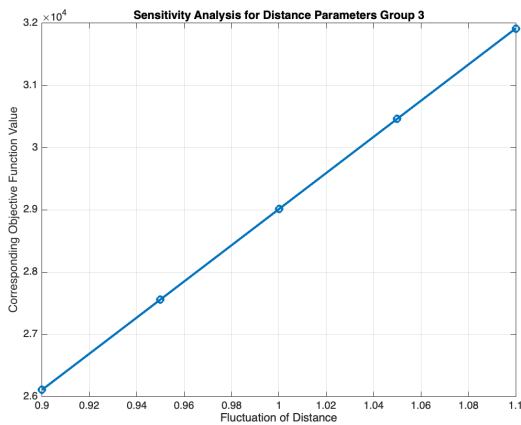


Figure 11 Sensitivity analysis Group 3

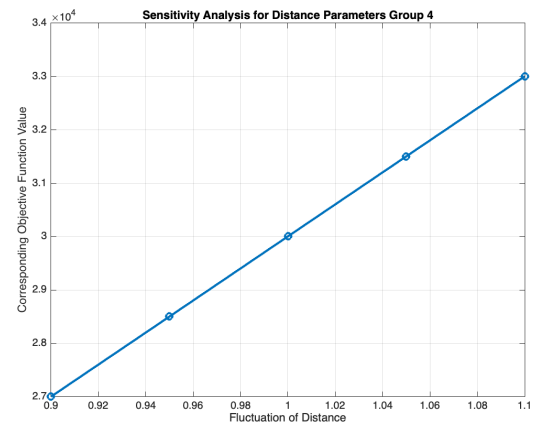


Figure 12 Sensitivity analysis Group 4

6.2.3 Analysis on score changes

The new scores were applied to the grouping functions, and the following results were obtained.

The objective value showed a linear increase as the the scores increased in the observed range. This indicates that the competitive variance between teams will be reduced if the performance of the teams are weakened, and vice versa. Making further optimization possible.

This will impact the exact grouping and schedule of the games. With a team's score lowering, it would be reallocated into a different group, changing the round robin groupings and therefore the opponents it will meet.

In reality, the performance of teams will be influenced by many factors. For example, the score of a team can improve as they continue to train and win more competitions, or deteriorate due to the injuries of key players. Therefore, the schedule need to consider possible change in these aspects for better flexibility.

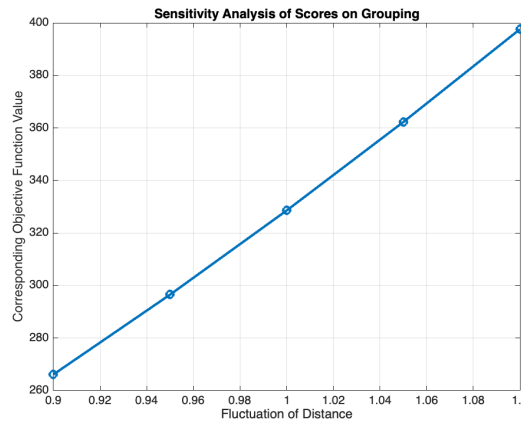


Figure 13 Sensitivity Analysis of Scores On Grouping

7 Model Evaluation

7.1 Strength and Weaknesses

Strength:

1. Sensitivity of our model is tested to ensure the credibility of results. Besides, the sensitivity model guarantees our model successfully addresses all targeted criteria
2. The model we built is objective and comprehensive enough to be used in real life situations. The schedule is easy to read and follow.
3. The model is easily accessible and readable, showing when each competition takes place.
4. The model is comprehensive, satisfying all components the IMMC looked for

Weakness:

1. The resting time is too short for some teams to recover compared to other currently existing international leagues
2. The attractive index of competition is relatively lower compared to other games, which might potentially impact the income

7.2 Conclusion

This study presents an optimized scheduling model for the Global Sports League (GSL) using nonlinear programming, genetic algorithms, and the Analytic Hierarchy Process (AHP). By balancing fairness, travel logistics, and competitive integrity, our model effectively addresses key scheduling challenges while ensuring economic viability and environmental sustainability.

Our results demonstrate the model's adaptability to league expansions and varying constraints, maintaining fair play and logistical feasibility. Sensitivity analyses confirm its robustness, highlighting key factors for optimization. This framework offers a practical and scalable solution for international sports scheduling, with potential applications across different leagues and sports.

References

- [1] Jeffery Stein and Jack Ransone. Game of basketball. *Sports Science Exchange*, 162, 2017. Accessed: 2025-03-10.
- [2] World Population Review. World population by country 2024 (live), 2024. Accessed: 2025-03-10.

Appendix A Code: Scheduling

```
1
2
3 % Create optimization variables
4 x12 = optimvar("x",5,5,10,"Type","integer","LowerBound",0,"UpperBound",1);
5
6 % Set initial starting point for the solver
7 initialPoint11.x = zeros(size(x12));
8
9 % Create problem
10 problem = optimproblem;
11
12 % Define problem objective
13 problem.Objective = fcn2optimexpr(@TimetableFairness_function,x12);
14
15 % Define problem constraints
16 problem.Constraints.constraint1 = cons_EqualHostAndGuest(x12);
17 problem.Constraints.constraint2 = cons_minimumRestTime(x12);
18 problem.Constraints.constraint3 = cons_NoSelfMatch(x12);
19 problem.Constraints.constraint4 = cons_RoundRobin(x12);
20
21 % Set nondefault solver options
22 options = optimoptions("ga","PlotFcn",["gaplotdistance","gaplotselection",...
23     "gaplotscorediversity","gaplotscores","gaplotstopping","gaplotbestf",...
24     "gaplotbestindiv"]);
25
26 % Display problem information
27 show(problem);
28
29 % Solve problem
30 [solution4,objectiveValue,reasonSolverStopped] = solve(problem,
31     initialPoint11,...
32     "Solver","ga","Options",options);
33
34 % Display results
35 solution4
36 reasonSolverStopped
37 objectiveValue
38
39 % Clear variables
40 clearvars x12 initialPoint11 options reasonSolverStopped objectiveValue
41
42 %Objective func
43 function obj = TimetableFairness_function(x)
44 %d = [0,14037,8916,10321,14171;
```

```
45 %14037,0,5359,7943,6776;
46 %8916,5359,0,6128,8430;
47 %10321,7943,6128,0,3975;
48 %14171,6776,8430,3975,0;];
49
50 %d = [0,8642,8217,16932,10780;
51 %8642,0,1482,8434,18597;
52 %8217,1482,0,8725,18985;
53 %16932,8434,8725,0,12300;
54 %10780,18597,18985,12300,0;];
55
56 %d = [0,6362,2794,11102,12813;
57 %6362,0,6007,9211,13274;
58 %2794,6007,0,8308,15597;
59 %11102,9211,8308,0,16082;
60 %12813,13274,15597,16082,0];
61
62 %d = [0,8700,3254,9352,12374;
63 %8700,0,9809,13777,17790;
64 %3254,9809,0,11253,10310;
65 %9352,13777,11253,0,8397;
66 %12374,17790,10310,8397,0;]
67
68 total_distance = 0;
69     for i=1:5
70         for j = 1:5
71             for k = 1:10
72                 total_distance = total_distance + d(i,j) * x(i,j,k);
73             end
74         end
75     end
76
77 intensityDifference = 0;
78 havegame = 0;
79     for i=1:5
80         for j = 1:5
81             for k = 1:10
82                 havegame = havegame + x(i,j,k) + x(j,i,k);
83                 intensityDifference = intensityDifference + (max(havegame) -
84                     min(havegame));
85             end
86         end
87     end
88 %
89
90 obj = 0.28571429 * total_distance + 0.71428571 * intensityDifference;
91
```

```
92 end
93
94 function cons = cons_minimumRestTime(x)
95     cons = [];
96     total_games = 0;
97     for i = 1:5
98         for k = 1:9
99             total_games = sum(x(i, :, k:k+1), 'all') + sum(x(:, i, k:k+1), '
100                 all');
101             cons = [cons; total_games <= 1];
102         end
103     end
104 end
105
106 function cons = cons_RoundRobin(x)
107     cons = [];
108     for i = 1:5
109         for j = i+1:5
110             cons = [cons; sum(x(i,j,:) + x(j,i,:)) == 1];
111         end
112     end
113 end
114
115 function cons = cons_NoSelfMatch(x)
116     cons = [];
117     for i = 1:5
118         for j = 1:5
119             for k = 1:10
120                 if i == j
121                     % Ensure no self-matches in any round
122                     cons = [cons; x(i, j, k) == 0];
123                 end
124             end
125         end
126     end
127 end
128
129 %roughly equal numbers for every team as host and as guest
130 function cons = cons_EqualHostAndGuest(x)
131     cons = [];
132     for i = 1:5
133         host = sum(x(i, :, :), 'all');
134         guest = sum(x(:, i, :), 'all');
135         cons = [cons; host == guest];
136     end
137 end
```

Appendix B Code: Grouping

```
1
2
3 % Create optimization variables
4 x13 = optimvar("x",20,4,"Type","integer","LowerBound",0,"UpperBound",1);
5
6 % Set initial starting point for the solver
7 initialPoint12.x = zeros(size(x13));
8
9 % Create problem
10 problem = optimproblem;
11
12 % Define problem objective
13 problem.Objective = fcn2optimexpr(@equitable_func,x13);
14
15 % Define problem constraints
16 problem.Constraints.constraint1 = cons_assignSingleGroup(x13);
17 problem.Constraints.constraint2 = cons_maxTeamsPerGroup(x13);
18 problem.Constraints.constraint3 = cons_DifferentContinents(x13);
19 problem.Constraints.constraint4 = cons_DifferentLevels(x13);
20
21 % Set nondefault solver options
22 options = optimoptions("ga","PlotFcn",["gaplotdistance","gaplotselection",...
23     "gaplotscorediversity","gaplotscores","gaplotstopping","gaplotbestf",...
24     "gaplotbestindiv"]);
25
26 % Display problem information
27 show(problem);
28
29 % Solve problem
30 [solution7,objectiveValue,reasonSolverStopped] = solve(problem,
31     initialPoint12,...
32     "Solver","ga","Options",options);
33
34 % Display results
35 solution7
36 reasonSolverStopped
37 objectiveValue
38
39 % Clear variables
40 clearvars x13 initialPoint12 options reasonSolverStopped objectiveValue
41
42 exact_values = solution.x;
43
44 % Display the exact values of the matrix
45 disp(exact_values);
```

```

45
46 function obj = equitable_func(x)
47     S = [52.85; 41.41; 40.94; 39.74; 48.85; 39.25; 38.22; 33.92; 76.11;
48         75.73;
49         75.34; 74.57; 84.01; 74.49; 60.69; 73.25; 67.29; 45.96; 73.31;
50         49.24];
51
52     group_competitiveness = sum(S .* x, 1); % Competitiveness per group (1x4
53         vector)
54     avg_competitiveness = mean(group_competitiveness);
55     variance = sum((group_competitiveness - avg_competitiveness).^2);
56
57     obj = variance;
58 end
59
60 function cons = cons_assignSingleGroup(x)
61     cons = optimconstr(20); % 20 constraints for 20 teams
62     for i = 1:20
63         cons(i) = sum(x(i,:)) == 1; % Each team in exactly one group
64     end
65 end
66
67 function cons = cons_maxTeamsPerGroup(x)
68     cons = optimconstr(4); % 4 constraints for 4 groups
69     for g = 1:4
70         cons(g) = sum(x(:,g)) == 5; % Each group has exactly 5 teams
71     end
72 end
73
74 function level_cons = cons_DifferentLevels(x)
75     % Define levels (L1=strongest, L5=weakest)
76     levels = [3; 4; 4; 4; 5; 3; 5; 5; 5; 1; 1; 1; 1; 2; 3; 2; 2; 4; 2; 3]
77
78     level_cons = optimconstr(0); % Initialize empty constraint
79     for g = 1:4
80         for i = 1:20
81             for j = i+1:20
82                 level_cons = [level_cons;
83                     x(i,g) + x(j,g) <= 1 + (levels(i) ~= levels(j))];
84             end
85         end
86     end
87 end
88
89 function continent_cons = cons_DifferentContinents(x)
90     continents = [1;1;1;1;1; 2;2;2;2; 3;3;3; 4;4;4; 5;5;5; 6;6];
91
92     continent_cons = optimconstr(0);

```

```
90     for g = 1:4
91         for i = 1:20
92             for j = i+1:20
93                 % Allow Asia (continent=1) to have 2 teams in one group
94                 if continents(i) == 1 && continents(j) == 1
95                     % Do not enforce the constraint for Asian teams
96                     continue;
97                 else
98                     continent_cons = [continent_cons;
99                                     x(i,g) + x(j,g) <= 1 + (continents(i) ~= continents(j))
100                                     ];
101             end
102         end
103     end
104 end
```